Aircraft Flight Dynamics
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MAE 331, Princeton University, 2018

Course Overview
Introduction to Flight Dynamics
Math Preliminaries

At the End of the Course, you should be able to:

- **Understand** aircraft configuration aerodynamics, performance, stability, and control
- **Estimate** an aircraft’s aerodynamic characteristics from geometric and inertial properties
- **Analyze** linear and nonlinear dynamic systems
- **Recognize** airplane modes of motion and their significance
- **Compute** aircraft motions
- **Appreciate** historical development of aviation
Syllabus, First Half

- Introduction, Math Preliminaries
- Point Mass Dynamics
- Aerodynamics of Airplane Configurations
  - Forces & Moments
  - 2-D & 3-D
  - Low- & High-Speed
- Cruising Flight Performance
  - Power & Thrust
  - Flight Envelope
- Gliding, Climbing, and Turning Performance
- Nonlinear, 6-DOF Equations of Motion
- Aircraft Control Devices and Systems

Details, reading, homework assignments, and references at http://blackboard.princeton.edu/

Syllabus, Second Half

- Linearized Equations of Motion
- Longitudinal Dynamics
- Lateral–Directional Dynamics
- Analysis of Linear Systems
  - Time Response
  - Transfer Functions and Frequency Response
  - Root Locus Analysis
- Flying Qualities Criteria
- Maneuvering at High Angles and Rates
- Aeroelasticity and Fuel Slosh
- Special Problems
You’re interested in MAE 331 because ...?

Details

- Lecture: 3-4:20, J-201, Tue & Thu, E-Quad
- Precept: 7:30-8:20, J-201, Mon
- Engineering, science, & math
- Case studies, historical context
- ~8 homework assignments
- Office hours: 1:30-2:30, MW, D-202, or any time my door is open. e-mail ahead, if possible
- Assistant in Instruction: Office hours: TBD
Details

- Lecture slides
  - PDFs from all 2016 lectures are available now at http://www.stengel.mypanel.princeton.edu/MAE331.html
  - PDF for current (2018) lecture on Blackboard morning of class or day before

- GRADING
  - Assignments: 50%
  - Term Paper: 30%
  - Class participation: 10%
  - Quick Quiz (10, 5 min): 10%

See: http://cte.virginia.edu/resources/grading-class-participation-2/

Text and References

- Science, Engineering, and Math:
  - Flight Dynamics, RFS, Princeton University Press, 2004

- Case Studies, Historical Context

- Technical Report PDFs on Blackboard

- Virtual reference book
**Flight Dynamics Book and Computer Code**

- Programs accessible from the *Flight Dynamics* web page
  - [http://www.princeton.edu/~stengel/FlightDynamics.html](http://www.princeton.edu/~stengel/FlightDynamics.html)
- ... or directly
- ERRATA for the book are listed there
- 6-degree-of-freedom nonlinear simulation of a business jet aircraft (MATLAB)
  - [http://www.princeton.edu/~stengel/FDcodeB.html](http://www.princeton.edu/~stengel/FDcodeB.html)
- Linear system analysis (MATLAB)
- Paper airplane simulation (MATLAB)
- Performance analysis of a business jet aircraft (Excel)
  - [http://www.princeton.edu/~stengel/Example261.xls](http://www.princeton.edu/~stengel/Example261.xls)

**Quick Quizzes**

*First 5 Minutes of 10 Classes*

- One question about the lectures and reading assignments from the previous week
- Largely qualitative but may require simple calculations
- Be sure to bring a pencil, paper, and calculator to class
Homework Assignments

- Groups of 2 or 3 students for all assignments
- Team members for each assignment will be different
- chosen using a spreadsheet and random number generator (TBD)
- Each member of each team will receive the same grade as the others
- Identify who did what on each assignment
- Submit via http://blackboard.princeton.edu/

Flight Tests Using Balsa Glider and Cockpit Flight Simulator

- In your Cessna 172:
  - Takeoff from Princeton Airport
  - Fly over Carnegie Lake
  - Land at Princeton Airport

- Compare actual flight of the glider with trajectory simulation
Assignment #1

- Document the physical characteristics and flight behavior of a balsa glider
  - Everything that you know about the physical characteristics
  - Everything that you know about the flight characteristics
- 2- or 3-person team, joint write-up
- Team assignments on http://blackboard.princeton.edu/

Luke Nash’s Biplane Glider
Flight #1 (MAE 331, 2008)
Frame Grab Sequence of Luke Nash’s Flight

- Can determine height, range, velocity, flight path angle, and pitch angle from sequence of digital photos

Trajectory Estimation from Photo Sequence

Smaller, fixed-interval time steps
Interpolation and differencing
Stability and Control Case Studies

F-100

Ercoupe

Electra

Reading Assignments

• Do Flight Dynamics reading before class

• Case Studies/Historical Context:

  **Airplane Stability and Control**
  • 10-minute synopses by groups of 3 students
    • Principal subject/scope of chapter
    • Technical ideas needed to understand chapter
    • When did the events occur?
    • 3 main “takeaway points” or conclusions
    • 3 most surprising or remarkable facts
Goals for Airplane Design

- Shape of airplane determined by purpose
- Safety, handling, performance, functioning, and comfort
- Agility vs. sedateness
- Control surfaces adequate to produce needed moments (i.e., torques)
- Tradeoffs, e.g., center of mass location
  - too far forward increases unpowered control-stick forces
  - too far aft degrades static stability

Configuration Driven By The Mission and Flight Envelope
Inhabited Air Vehicles

Uninhabited Air Vehicles (UAV)
Introduction to Flight Dynamics

Airplane Components
Airplane Rotational Degrees of Freedom

Airplane Translational Degrees of Freedom
Phases of Flight

Flight of a Paper Airplane
Flight of a Paper Airplane
Example 1.3-1, Flight Dynamics

- Equations of motion integrated numerically to estimate the flight path
- **Red**: Equilibrium flight path
- **Black**: Initial flight path angle = 0
- **Blue**: plus increased initial airspeed
- **Green**: loop

![Graph showing flight dynamics](image)
Assignment #2

- Compute the trajectory of a balsa glider
- Simulate using equations of motion
- Compare to the actual flight of the glider (HW #1)
- Similar to the flight of a paper airplane

Gliding Flight
Configuration Aerodynamics

(b) Glide aerodynamic characteristics.

Math Preliminaries
Dynamic Systems

Dynamic Process: Current state depends on prior state
- \( x \) = dynamic state
- \( u \) = input
- \( w \) = exogenous disturbance
- \( p \) = parameter
- \( t \) or \( k \) = time or event index

Observation Process: Measurement may contain error or be incomplete
- \( y \) = output (error-free)
- \( z \) = measurement
- \( n \) = measurement error

All of these quantities are multidimensional. They can be expressed as vectors.

Notation for Scalars and Vectors

- **Scalar**: usually lower case: \( a, b, c, \ldots, x, y, z \)

  \[
  a = 12; \quad b = 7; \quad c = a + b = 19; \quad x = a + b^2 = 12 + 49 = 61
  \]

- **Vector**: usually bold or with underbar: \( \mathbf{x} \) or \( \underline{x} \)
  - Ordered set
  - Column of scalars
  - Dimension = \( n \times 1 \)

\[
\mathbf{a} = \begin{bmatrix} 2 \\ -7 \\ 16 \end{bmatrix}; \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}; \quad \mathbf{y} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}
\]
**Matrices and Transpose**

- **Matrix**: usually bold capital or capital: $F$ or $F$
  - Dimension $= (m \times n)$

\[
\begin{align*}
x &= \begin{bmatrix} p \\ q \\ r \end{bmatrix}; \quad A &= \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \\ l & m & n \end{bmatrix}
\end{align*}
\]

- **Transpose**: interchange rows and columns

\[
\begin{align*}
x^T &= \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \\
A^T &= \begin{bmatrix} a & d & g & l \\ b & e & h & m \\ c & f & k & n \end{bmatrix}
\end{align*}
\]

**Multiplication**

- Operands must be conformable
- Multiplication of vector by scalar is associative, commutative, and distributive

\[
\begin{align*}
ax &= xa = \begin{bmatrix} ax_1 \\ ax_2 \\ ax_3 \end{bmatrix} \\
a(x + y) &= (x + y)a = (ax + ay)
\end{align*}
\]

\[
dim(x) = dim(y)
\]

\[
ax^T = \begin{bmatrix} ax_1 & ax_2 & ax_3 \end{bmatrix}
\]
Addition

- Conformable vectors and matrices are added term by term

\[
x = \begin{bmatrix} a \\ b \end{bmatrix} \quad ; \quad z = \begin{bmatrix} c \\ d \end{bmatrix}
\]

\[
x + z = \begin{bmatrix} a + c \\ b + d \end{bmatrix}
\]

Inner Product

- Inner (dot) product of vectors produces scalar result

\[
x^T x = x \cdot x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}
\]

\[
= (x_1^2 + x_2^2 + x_3^2)
\]

- Length (or magnitude) of vector is square root of dot product

\[
= (x_1^2 + x_2^2 + x_3^2)^{1/2}
\]
Vector Transformation

- Matrix-vector product transforms one vector into another
- Matrix-matrix product produces a new matrix

\[ y = Ax = \begin{bmatrix} 2 & 4 & 6 \\ 3 & -5 & 7 \\ 4 & 1 & 8 \\ -9 & -6 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \]

\((n \times 1) = (n \times m)(m \times 1)\)

Derivatives and Integrals of Vectors

Derivatives and integrals of vectors are vectors of derivatives and integrals

\[ \frac{dx}{dt} = \begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \frac{dx_3}{dt} \end{bmatrix} \quad \int x \, dt = \begin{bmatrix} \int x_1 \, dt \\ \int x_2 \, dt \\ \int x_3 \, dt \end{bmatrix} \]
Matrix Inverse

**Transformation** 
\[ x_2 = A x_1 \]

\[
\begin{bmatrix}
\text{x} \\
\text{y} \\
\text{z}
\end{bmatrix}_2 =
\begin{bmatrix}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{bmatrix}
\begin{bmatrix}
\text{x} \\
\text{y} \\
\text{z}
\end{bmatrix}_1
\]

**Inverse Transformation** 
\[ x_1 = A^{-1} x_2 \]

\[
\begin{bmatrix}
\text{x} \\
\text{y} \\
\text{z}
\end{bmatrix}_1 =
\begin{bmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{bmatrix}
\begin{bmatrix}
\text{x} \\
\text{y} \\
\text{z}
\end{bmatrix}_2
\]

Matrix Identity and Inverse

- **Identity matrix**: no change when it multiplies a conformable vector or matrix

\[
I_3 =
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[ y = I y \]

- **A non-singular square matrix** multiplied by its inverse forms an identity matrix

\[
A A^{-1} = A^{-1} A = I
\]

\[
A A^{-1} =
\begin{bmatrix}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{bmatrix}
\begin{bmatrix}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{bmatrix}
\]

\[
= \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Mathematical Models of Dynamic Systems are Differential Equations

Continuous-time dynamic process: Vector Ordinary Differential Equation

\[
\dot{x}(t) \doteq \frac{dx(t)}{dt} = f[x(t), u(t), w(t), p(t), t]
\]

Output Transformation

\[
y(t) = h[x(t), u(t)]
\]

Measurement with Error

\[
z(t) = y(t) + n(t)
\]

Next Time:

Point-Mass Dynamics
Aerodynamic/Thrust Forces
Supplemental Material

Helpful Resources

- Web pages
  - http://blackboard.princeton.edu/
  - http://www.stengel.mycpanel.princeton.edu/MAE331.html
- Princeton University Engineering Library (paper and on-line)
- NACA/NASA pubs
  - http://ntrs.nasa.gov/search.jsp
## MAE 331 Course Learning Objectives

*(Accreditation Board for Engineering and Technology)*

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<thead>
<tr>
<th>Course Learning Objectives</th>
<th>ABET Criterion 3</th>
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<tbody>
<tr>
<td>Understanding of the dynamics and control of aircraft.</td>
<td>a</td>
</tr>
<tr>
<td>Ability to estimate aerodynamic coefficients and stability derivatives from aircraft geometry and flight envelope.</td>
<td>a, c</td>
</tr>
<tr>
<td>Facility in analyzing mathematical descriptions of the rigid-body motions of flying vehicles.</td>
<td>a</td>
</tr>
<tr>
<td>Ability to estimate the performance, stability, and control characteristics of aircraft.</td>
<td>b</td>
</tr>
<tr>
<td>Development of appreciation for flight-testing methods and results.</td>
<td>b, k</td>
</tr>
<tr>
<td>Ability to apply systems-engineering approach to the analysis, design, and testing of aircraft.</td>
<td>b, c</td>
</tr>
<tr>
<td>Demonstration of ability to work in multidisciplinary teams.</td>
<td>d</td>
</tr>
<tr>
<td>Demonstration of computational problem-solving, through thorough knowledge, application, and development of analytical software.</td>
<td>e, k</td>
</tr>
<tr>
<td>Appreciation of the historical context within which airplanes have evolved to present-day configurations.</td>
<td>f, h, i, j</td>
</tr>
<tr>
<td>Competence in presenting ideas.</td>
<td>g</td>
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