Aircraft Equations of Motion: Translation and Rotation
Robert Stengel, Aircraft Flight Dynamics, MAE 331, 2018

Learning Objectives
• What use are the equations of motion?
• How is the angular orientation of the airplane described?
• What is a cross-product-equivalent matrix?
• What is angular momentum?
• How are the inertial properties of the airplane described?
• How is the rate of change of angular momentum calculated?

Reading:
Flight Dynamics
155–161

Review Questions
• What characteristic(s) provide maximum gliding range?
• Do gliding heavy airplanes fall out of the sky faster than light airplanes?
• Are the factors for maximum gliding range and minimum sink rate the same?
• How does the maximum climb rate vary with altitude?
• What are “energy height” and “specific excess power”?
• What is an “energy climb”?
• How is the “maneuvering envelope” defined?
• What factors determine the maximum steady turning rate?
### Dynamic Systems

**Dynamic Process:** Current state depends on prior state

- \( x \): dynamic state
- \( u \): input
- \( w \): exogenous disturbance
- \( p \): parameter
- \( t \) or \( k \): time or event index

**Observation Process:** Measurement may contain error or be incomplete

- \( y \): output (error-free)
- \( z \): measurement
- \( n \): measurement error

\[
\frac{dx(t)}{dt} = f(x(t), u(t), w(t), p(t), t)
\]

\[
y(t) = h[x(t), u(t)]
\]

\[
z(t) = y(t) + n(t)
\]

### Ordinary Differential Equations

Ordinary Differential Equations Fall Into 4 Categories

- Nonlinear, time-varying models (NTV)
- Nonlinear, time-invariant models (NTI)
- Linear, time-varying models (LTV)
- Linear, time-invariant models (LTI)

\[
\frac{dx(t)}{dt} = f(x(t), u(t), w(t), p(t), t)
\]

\[
\frac{dx(t)}{dt} = F(t)x(t) + G(t)u(t) + L(t)w(t)
\]

\[
\frac{dx(t)}{dt} = f(x(t), u(t), w(t))
\]

\[
\frac{dx(t)}{dt} = Fx(t) + Gu(t) + Lw(t)
\]
What Use are the Equations of Motion?

- Nonlinear equations of motion
  - Compute “exact” flight paths and motions
    - Simulate flight motions
    - Optimize flight paths
    - Predict performance
  - Provide basis for approximate solutions

\[
\frac{dx(t)}{dt} = f[x(t), u(t), w(t), p(t), t] \]

\[
\frac{dx(t)}{dt} = F x(t) + G u(t) + L w(t) \]

- Linear equations of motion
  - Simplify computation of flight paths and solutions
  - Define modes of motion
  - Provide basis for control system design and flying qualities analysis

Examples of Airplane Dynamic System Models

- Nonlinear, Time-Varying
  - Large amplitude motions
  - Significant change in mass

- Linear, Time-Varying
  - Small amplitude motions
  - Perturbations from a dynamic flight path

- Nonlinear, Time-Invariant
  - Large amplitude motions
  - Negligible change in mass

- Linear, Time-Invariant
  - Small amplitude motions
  - Perturbations from an equilibrium flight path
Translational Position

Position of a Particle

Projections of vector magnitude on three axes

\[ \mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = r \begin{bmatrix} \cos \delta_x \\ \cos \delta_y \\ \cos \delta_z \end{bmatrix} \]

\[ \begin{bmatrix} \cos \delta_x \\ \cos \delta_y \\ \cos \delta_z \end{bmatrix} = \text{Direction cosines} \]
Cartesian Frames of Reference

- Two reference frames of interest
  - \( I \): Inertial frame (fixed to inertial space)
  - \( B \): Body frame (fixed to body)

- Translation
  - Relative linear positions of origins

- Rotation
  - Orientation of the body frame with respect to the inertial frame

Measurement of Position in Alternative Frames - 1

- Two reference frames of interest
  - \( I \): Inertial frame (fixed to inertial space)
  - \( B \): Body frame (fixed to body)

\[
\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}
\]

\[
\mathbf{r}_{\text{particle}} = \mathbf{r}_{\text{origin}} + \Delta\mathbf{r}_{\text{w.r.t.origin}}
\]

- Differences in frame orientations must be taken into account in adding vector components
Measurement of Position in Alternative Frames - 2

Inertial-axis view
\[
\mathbf{r}_{\text{particle}_i} = \mathbf{r}_{\text{origin-}B_i} + \mathbf{H}_B^I \Delta \mathbf{r}_B
\]

Body-axis view
\[
\mathbf{r}_{\text{particle}_B} = \mathbf{r}_{\text{origin-}I_B} + \mathbf{H}_I^B \Delta \mathbf{r}_I
\]
Direction Cosine Matrix

\[ H_{I}^{B} = \begin{bmatrix} \cos \delta_{11} & \cos \delta_{21} & \cos \delta_{31} \\ \cos \delta_{12} & \cos \delta_{22} & \cos \delta_{32} \\ \cos \delta_{13} & \cos \delta_{23} & \cos \delta_{33} \end{bmatrix} \]

- Projections of unit vector components of one reference frame on another
- Rotational orientation of one reference frame with respect to another
- Cosines of angles between each I axis and each B axis

Properties of the Rotation Matrix

\[ H_{I}^{B} = \begin{bmatrix} \cos \delta_{11} & \cos \delta_{21} & \cos \delta_{31} \\ \cos \delta_{12} & \cos \delta_{22} & \cos \delta_{32} \\ \cos \delta_{13} & \cos \delta_{23} & \cos \delta_{33} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \]

\[ r_{B} = H_{I}^{B} r_{I} \quad s_{B} = H_{I}^{B} s_{I} \]

Orthonormal transformation

Angles between vectors are preserved

Lengths are preserved

\[ |r_{I}| = |r_{B}| \quad ; \quad |s_{I}| = |s_{B}| \]

\[ \angle(r_{I}, s_{I}) = \angle(r_{B}, s_{B}) = x \text{ deg} \]
Euler Angles

- Body attitude measured with respect to inertial frame
- Three-angle orientation expressed by sequence of three orthogonal single-angle rotations

\[ \text{Inertial} \Rightarrow \text{Intermediate}_1 \Rightarrow \text{Intermediate}_2 \Rightarrow \text{Body} \]

- 24 ($\pm 12$) possible sequences of single-axis rotations
- Aircraft convention: 3-2-1, z positive down

$\psi$: Yaw angle
$\theta$: Pitch angle
$\phi$: Roll angle

Euler Angles Measure the Orientation of One Frame with Respect to the Other

- Conventional sequence of rotations from inertial to body frame
  - Each rotation is about a single axis
  - Right-hand rule
  - Yaw, then pitch, then roll
  - These are called Euler Angles

Other sequences of 3 rotations can be chosen; however, once sequence is chosen, it must be retained
Reference Frame Rotation from Inertial to Body: Aircraft Convention (3-2-1)

### Yaw rotation ($\phi$) about $z_i$ axis

$$
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}
= 
\begin{bmatrix}
  \cos \psi & \sin \psi & 0 \\
  -\sin \psi & \cos \psi & 0 \\
  0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  x_i \\
  y_i \\
  z_i
\end{bmatrix}
= 
\begin{bmatrix}
  x \cos \psi + y \sin \psi \\
  -x \sin \psi + y \cos \psi \\
  z
\end{bmatrix}
$$

$$r_1 = H_1 r$$

### Pitch rotation ($\theta$) about $y_i$ axis

$$
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}
= 
\begin{bmatrix}
  \cos \theta & 0 & -\sin \theta \\
  0 & 1 & 0 \\
  \sin \theta & 0 & \cos \theta
\end{bmatrix}
\begin{bmatrix}
  x_i \\
  y_i \\
  z_i
\end{bmatrix}
= 
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}
$$

$$r_2 = H_2 r_1 = H_2 H_1 r$$

### Roll rotation ($\phi$) about $x_i$ axis

$$
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}
= 
\begin{bmatrix}
  1 & 0 & 0 \\
  0 & \cos \phi & \sin \phi \\
  0 & -\sin \phi & \cos \phi
\end{bmatrix}
\begin{bmatrix}
  x_i \\
  y_i \\
  z_i
\end{bmatrix}
= 
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}
$$

$$r_3 = H_3 r_2 = H_3 H_2 H_1 r$$

The Rotation Matrix

The three-angle rotation matrix is the product of 3 single-angle rotation matrices:

$$H_3^i(\phi, \theta, \psi) = H_3^i(\phi) H_2^i(\theta) H_1^i(\psi)$$

$$
\begin{bmatrix}
  1 & 0 & 0 \\
  0 & \cos \phi & \sin \phi \\
  0 & -\sin \phi & \cos \phi
\end{bmatrix}
\begin{bmatrix}
  \cos \theta & 0 & -\sin \theta \\
  0 & 1 & 0 \\
  \sin \theta & 0 & \cos \theta
\end{bmatrix}
\begin{bmatrix}
  \cos \psi & \sin \psi & 0 \\
  -\sin \psi & \cos \psi & 0 \\
  0 & 0 & 1
\end{bmatrix}
$$

$$= 
\begin{bmatrix}
  \cos \theta \cos \psi & \cos \theta \sin \psi & -\sin \theta \\
  -\cos \phi \sin \psi + \sin \phi \sin \theta \cos \psi & \cos \phi \cos \psi + \sin \phi \sin \theta \sin \psi & \sin \phi \cos \theta \\
  \sin \phi \sin \psi + \cos \phi \sin \theta \cos \psi & -\sin \phi \cos \psi + \cos \phi \sin \theta \sin \psi & \cos \phi \cos \theta
\end{bmatrix}
$$

an expression of the Direction Cosine Matrix
Rotation Matrix Inverse

Inverse relationship: interchange sub- and superscripts

\[
\mathbf{r}_B = \mathbf{H}_I^B \mathbf{r}_I
\]

\[
\mathbf{r}_I = (\mathbf{H}_I^B)^{-1} \mathbf{r}_B = \mathbf{H}_B^I \mathbf{r}_B
\]

Because transformation is **orthonormal**

Inverse = transpose

Rotation matrix is always **non-singular**

\[
\begin{bmatrix}
\mathbf{H}_I^B (\phi, \theta, \psi)
\end{bmatrix}^{-1} = \begin{bmatrix}
\mathbf{H}_I^B (\phi, \theta, \psi)
\end{bmatrix}^T = \mathbf{H}_B^I (\psi, \theta, \phi)
\]

\[
\mathbf{H}_B^I = (\mathbf{H}_I^B)^{-1} = (\mathbf{H}_I^B)^T = \mathbf{H}_1^I \mathbf{H}_2^I \mathbf{H}_B^2
\]

\[
\mathbf{H}_B^I \mathbf{H}_I^B = \mathbf{H}_I^B \mathbf{H}_B^I = \mathbf{I}
\]

Checklist

- What are direction cosines?
- What are Euler angles?
- What rotation sequence is used to describe airplane attitude?
- What are properties of the rotation matrix?
Angular Momentum of a Particle

- Moment of linear momentum of differential particles that make up the body
  - (Differential masses) x components of the velocity that are perpendicular to the moment arms

\[
dh = ( \mathbf{r} \times dm \mathbf{v} ) = ( \mathbf{r} \times \mathbf{v}_m ) dm
\]

\[
= \left[ \mathbf{r} \times ( \mathbf{v}_o + \mathbf{\omega} \times \mathbf{r} ) \right] dm
\]

- Cross Product: Evaluation of a determinant with unit vectors \((i, j, k)\) along axes, \((x, y, z)\) and \((v_x, v_y, v_z)\) projections on axes

\[
\mathbf{r} \times \mathbf{v} = \begin{vmatrix} i & j & k \\ x & y & z \\ v_x & v_y & v_z \end{vmatrix} = (yv_z - zv_y)i + (zv_x - xv_z)j + (xv_y - yv_x)k
\]
Cross-Product-Equivalent Matrix

\[
\mathbf{r} \times \mathbf{v} = \begin{bmatrix} i & j & k \\ x & y & z \\ v_x & v_y & v_z \end{bmatrix} = (yv_z - zv_y)i + (zv_x - xv_z)j + (xv_y - yv_x)k
\]

\[
= \begin{bmatrix} yv_z - zv_y \\ zv_x - xv_z \\ xv_y - yv_x \end{bmatrix} = \hat{\mathbf{r}} \mathbf{v} = \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}
\]

Angular Momentum of the Aircraft

- Integrate moment of linear momentum of differential particles over the body

\[
\mathbf{h} = \int_{\text{Body}} \mathbf{r} \times (\mathbf{v} + \omega \times \mathbf{r}) \, dm = \int_{x_{\text{min}}}^{x_{\text{max}}} \int_{y_{\text{min}}}^{y_{\text{max}}} \int_{z_{\text{min}}}^{z_{\text{max}}} (\mathbf{r} \times (\mathbf{\omega} \times \mathbf{r})) \rho(x,y,z) \, dx \, dy \, dz = \begin{bmatrix} h_x \\ h_y \\ h_z \end{bmatrix}
\]

\[
\rho(x,y,z) = \text{Density of the body}
\]

- Choose the center of mass as the rotational center

\[
\mathbf{h} = \int_{\text{Body}} (\mathbf{r} \times \mathbf{v}) \, dm + \int_{\text{Body}} (\mathbf{r} \times (\mathbf{\omega} \times \mathbf{r})) \, dm = 0 - \int_{\text{Body}} (\mathbf{r} \times (\mathbf{r} \times \mathbf{\omega})) \, dm = - \int_{\text{Body}} (\mathbf{r} \times \mathbf{r}) \, dm \times \mathbf{\omega} = - \int_{\text{Body}} \left( \hat{\mathbf{r}} \hat{\mathbf{r}} \right) \, dm \mathbf{\omega}
\]
Location of the Center of Mass

\[
\mathbf{r}_{cm} = \frac{1}{m} \int_{\text{Body}} \mathbf{r} \, dm = \frac{1}{m} \int_{x_{\text{min}}}^{x_{\text{max}}} \int_{y_{\text{min}}}^{y_{\text{max}}} \int_{z_{\text{min}}}^{z_{\text{max}}} \mathbf{r} \rho(x, y, z) \, dx \, dy \, dz = \begin{bmatrix}
x_{cm} \\
y_{cm} \\
z_{cm}
\end{bmatrix}
\]

The Inertia Matrix
The Inertia Matrix

\[
h = - \int \dot{\mathbf{r}} \mathbf{r} \, \omega \, dm = - \int \dot{\mathbf{r}} \mathbf{r} \, dm = \mathbf{I} \omega
\]

\[
\mathbf{I} = - \int \dot{\mathbf{r}} \mathbf{r} \, dm = - \int \dot{\mathbf{r}} \mathbf{r} \, dm = \int \begin{bmatrix}
0 & -z & y \\
z & 0 & -x \\
-y & x & 0
\end{bmatrix} dm
\]

\[
= \int \begin{bmatrix}
(y^2 + z^2) & -xy & -xz \\
-xy & (x^2 + z^2) & -yz \\
-xz & -yz & (x^2 + y^2)
\end{bmatrix} dm
\]

**Inertia matrix** derives from equal effect of angular rate on all particles of the aircraft.

Moments and Products of Inertia

\[
\mathbf{I} = \int \begin{bmatrix}
(y^2 + z^2) & -xy & -xz \\
-xy & (x^2 + z^2) & -yz \\
-xz & -yz & (x^2 + y^2)
\end{bmatrix} dm = \begin{bmatrix}
I_{xx} & -I_{xy} & -I_{xz} \\
-I_{xy} & I_{yy} & -I_{yz} \\
-I_{xz} & -I_{yz} & I_{zz}
\end{bmatrix}
\]

**Inertia matrix**

- Moments of inertia on the diagonal
- Products of inertia off the diagonal

- If products of inertia are zero, \((x, y, z)\) are principal axes $\rightarrow$
- All rigid bodies have a set of principal axes

**Ellipsoid of Inertia**

\[
I_{xx}x^2 + I_{yy}y^2 + I_{zz}z^2 = 1
\]
Inertia Matrix of an Aircraft with Mirror Symmetry

\[ \mathbf{I} = \int_{\text{Body}} \begin{bmatrix} (y^2 + z^2) & 0 & -xz \\ 0 & (x^2 + z^2) & 0 \\ -xz & 0 & (x^2 + y^2) \end{bmatrix} \, dm = \begin{bmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{xz} & 0 & I_{zz} \end{bmatrix} \]

Nose high/low product of inertia, \( I_{xz} \)

Nominal Configuration
Tips folded, 50% fuel, \( W = 38,524 \text{ lb} \)
\[
\begin{align*}
\mathbf{x}_{\text{cm}} @ 0.218 \tau \\
I_{xx} &= 1.8 \times 10^6 \text{ slug-ft}^2 \\
I_{yy} &= 19.9 \times 10^6 \text{ slug-ft}^2 \\
I_{zz} &= 22.1 \times 10^6 \text{ slug-ft}^2 \\
I_{xz} &= -0.88 \times 10^6 \text{ slug-ft}^2
\end{align*}
\]

Checklist

- How is the location of the center of mass found?
- What is a cross-product-equivalent matrix?
- What is the inertia matrix?
- What is an ellipsoid of inertia?
- What does the “nose-high” product of inertia represent?
**Historical Factoids**

**Technology of World War II Aviation**

- **1938-45:** Analytical and experimental approach to design
  - Many configurations designed and flight-tested
  - Increased specialization; radar, navigation, and communication
  - Approaching the “sonic barrier”

- **Aircraft Design**
  - Large, powerful, high-flying aircraft
  - Turbocharged engines
  - Oxygen and Pressurization

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**Power Effects on Stability and Control**

- **Brewster Buffalo:** over-armored and under-powered
- During W.W.II, the size of fighters remained about the same, but installed horsepower doubled (*F4F* vs. *F8F*)
- Use of flaps means high power at low speed, increasing relative significance of thrust effects
World War II Carrier-Based Airplanes

- Takeoff without catapult, relatively low landing speed [http://www.youtube.com/watch?v=4dySbhKTVNk](http://www.youtube.com/watch?v=4dySbhKTVNk)
- Tailhook and arresting gear
- Carrier steams into wind
- Design for storage (short tail length, folding wings) affects stability and control

Multi-Engine Aircraft of World War II

- Large W.W.II aircraft had unpowered controls:
  - High foot-pedal force
  - Rudder stability problems arising from balancing to reduce pedal force
- Severe engine-out problem for twin-engine aircraft
WW II Military Flying Boats

Seaplanes proved useful during World War II

- Lockheed PBY Catalina
- Martin PBM Mariner
- Boeing XPBB Sea Ranger
- Saunders-Roe SR.35 Lerwick
- Grumman JRF-1 Goose

Rate of Change of Angular Momentum
Newton’s 2nd Law, Applied to Rotational Motion

In inertial frame, rate of change of angular momentum = applied moment (or torque), \( M \)

\[
\frac{d\mathbf{h}}{dt} = \frac{d(I\mathbf{\omega})}{dt} = I\frac{d\mathbf{\omega}}{dt} + I\mathbf{\omega} \frac{dI}{dt} = I\mathbf{\dot{\omega}} + I\mathbf{\omega} = \mathbf{M} = \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix}
\]

Angular Momentum and Rate

Angular momentum and rate vectors are not necessarily aligned

\( \mathbf{h} = I\mathbf{\omega} \)
How Do We Get Rid of $\frac{dI}{dt}$ in the Angular Momentum Equation?

**Chain Rule**  
... and in an inertial frame

$$\frac{d(I\omega)}{dt} = \dot{I}\omega + I\dot{\omega}$$

\[\dot{I} \neq 0\]

- **Dynamic equation in a body-referenced frame**
  - Inertial properties of a constant-mass, rigid body are **unchanging** in a body frame of reference
  - ... but a body-referenced frame is "non-Newtonian" or "non-inertial"
  - Therefore, dynamic equation must be **modified** for expression in a rotating frame

---

Angular Momentum Expressed in Two Frames of Reference

- **Angular momentum and rate** are **vectors**
  - Expressed in either the inertial or body frame
  - Two frames related algebraically by the **rotation matrix**

\[
\begin{align*}
    h_B(t) &= \mathbf{H}_I^B(t) h_I(t); & h_I(t) &= \mathbf{H}_B^I(t) h_B(t) \\
    \omega_B(t) &= \mathbf{H}_I^B(t) \omega_I(t); & \omega_I(t) &= \mathbf{H}_B^I(t) \omega_B(t)
\end{align*}
\]
Vector Derivative Expressed in a Rotating Frame

Chain Rule
\[ \dot{\mathbf{h}}_I = H_B^I \dot{h}_B + \ddot{H}_B^I h_B \]

Alternatively
\[ \dot{\mathbf{h}}_I = H_B^I \dot{h}_B + \omega_I \times \mathbf{h}_I = H_B^I \dot{h}_B + \tilde{\omega}_I \mathbf{h}_I \]

Consequently, the 2nd term is
\[ \ddot{H}_B^I h_B = \tilde{\omega}_I \mathbf{h}_I = \tilde{\omega}_I H_B^I h_B \]

... where the cross-product equivalent matrix of angular rate is
\[ \tilde{\omega} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \]

---

External Moment Causes Change in Angular Rate

Positive rotation of Frame B w.r.t. Frame A is a negative rotation of Frame A w.r.t. Frame B

In the body frame of reference, the angular momentum change is
\[ \dot{h}_B = H_B^I \dot{h}_I + H_I^B \dot{h}_I = H_B^I \dot{h}_I - \omega_B \times h_B = H_B^I \dot{h}_I - \tilde{\omega}_B h_B \]
\[ = H_B^I M_I - \tilde{\omega}_B \mathbf{I}_B \omega_B = M_B - \tilde{\omega}_B \mathbf{I}_B \omega_B \]

Moment = torque = force \times moment arm
\[ M_I = \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} ; \quad M_B = H_B^I M_I = \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} \begin{bmatrix} L \\ M \\ N \end{bmatrix} \]
Rate of Change of Body-Referenced Angular Rate due to External Moment

In the body frame of reference, the angular momentum change is

\[
\dot{h}_B = H_B^{\dot{h}} i_B + H_B^{\dot{h}} h_B = H_B^{\dot{h}} i_B - \omega_B \times h_B
\]

\[
= H_B^{\dot{h}} i_B - \tilde{\omega}_B h_B = H_B^{\dot{h}} M - \tilde{\omega}_B I_B \omega_B
\]

\[
= M_B - \tilde{\omega}_B I_B \omega_B
\]

For constant body-axis inertia matrix

\[
\dot{h}_B = I_B \dot{\omega}_B = M_B - \tilde{\omega}_B I_B \omega_B
\]

Consequently, the differential equation for angular rate of change is

\[
\dot{\omega}_B = I_B^{-1} \left( M_B - \tilde{\omega}_B I_B \omega_B \right)
\]

Checklist

- Why is it inconvenient to solve momentum rate equations in an inertial reference frame?
- Are angular rate and momentum vectors aligned?
- How are angular rate equations transformed from an inertial to a body frame?
Learning Objectives

- How is a rotating reference frame described in an inertial reference frame?
- Is the transformation singular?
- What adjustments must be made to expressions for forces and moments in a non-inertial frame?
- How are the 6-DOF equations implemented in a computer?
- Damping effects
Moments and Products of Inertia

Moments and products of inertia tabulated for geometric shapes with uniform density

Construct aircraft moments and products of inertia from components using parallel-axis theorem

Model in CREO, etc.

Moments and Products of Inertia

(Bedford & Fowler)

Moments and products of inertia tabulated for geometric shapes with uniform density

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