Maneuvering at High Angles and Angular Rates
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MAE 331, 2018

Learning Objectives

• High angle of attack and angular rates
• Asymmetric flight
• Nonlinear aerodynamics
• Inertial coupling
• Spins and tumbling

Tactical Airplane Maneuverability

• Maneuverability parameters
  – Stability
  – Roll rate and acceleration
  – Normal load factor
  – Thrust/weight ratio
  – Pitch rate
  – Transient response
  – Control forces

• Dogfights
  – Preferable to launch missiles at long range
  – Dogfight is a backup tactic
  – Preferable to have an unfair advantage

• Air-combat sequence
  – Detection
  – Closing
  – Attack
  – Maneuvers, e.g.,
    – Scissors
    – High yo-yo
  – Disengagement
Coupling of Longitudinal and Lateral-Directional Motions

Longitudinal Motions can Couple to Lateral-Directional Motions

- Linearized equations have limited application to high-angle/high-rate maneuvers
  - Steady, non-zero sideslip angle (Sec. 7.1, FD)
  - Steady turn (Sec. 7.1, FD)
  - Steady roll rate

\[
\mathbf{F} = \begin{bmatrix}
F_{\text{Lon}} & F_{\text{Lon-Dir}} \\
F_{\text{Lat-Dir}} & F_{\text{Lat-Dir}}
\end{bmatrix}
\]

\[F_{\text{Lon-Dir}} \neq 0\]
Stability Boundaries Arising From Asymmetric Flight

Stability Boundaries with Nominal Sideslip, $\beta_o$, and Roll Rate, $p_o$
Pitch-Yaw Coupling Due To Steady Roll Rate, $p_o$

- Combine 2nd-order short period and Dutch roll modes
  - Body axes
  - Constant roll rate = $p_o$, rad/s

State vector

$$\Delta x(t) = \begin{bmatrix} \Delta x_{low} \\ \Delta x_{LD} \end{bmatrix} = \begin{bmatrix} \Delta w \\ \Delta q \\ \Delta v \\ \Delta r \end{bmatrix}$$

- Normal velocity, m/s
- Pitch rate, rad/s
- Side velocity, m/s
- Yaw rate, rad/s

Control input vector

$$\Delta u(t) = \begin{bmatrix} \Delta \delta E \\ \Delta \delta A \\ \Delta \delta R \end{bmatrix}$$

- Elevator, deg or rad
- Ailerons, deg or rad
- Rudder, deg or rad

4th-order dynamic model

$$\begin{bmatrix} \Delta x_{low} \\ \Delta x_{LD} \end{bmatrix} = \begin{bmatrix} F_{low} & F_{LD}^L \\ F_{LD}^H & F_{LD} \end{bmatrix} \begin{bmatrix} \Delta x_{low} \\ \Delta x_{LD} \end{bmatrix} + \begin{bmatrix} G_{low} \\ G_{LD} \end{bmatrix} \Delta u$$

Time Response to Elevator Step Input

- When $p_o = 0^\circ$/s
  - Elevator input produces longitudinal response but no lateral-directional response

- At $p_o = 60^\circ$/s
  - Short-period (faster) mode dominates longitudinal response
  - Dutch-roll (slower) mode dominates lateral directional response

- At $p_o = 120^\circ$/s
  - Both modes are evident in both responses
  - Fast mode is even faster
  - Slow mode is even slower
**Pitch-Yaw Coupling Due To Steady Roll Rate, \( p_o \)**

- 4th-order stability matrix
  - Body axes
  - Negligible \( v_o, u_o \sim V_N \)
  - Negligible coupling aerodynamic effects
- Constant roll rate is only source of coupling

\[
\begin{bmatrix}
F_{Lw} & F_{Lw}^{LD} \\
F_{Lw} & F_{LD}
\end{bmatrix} =
\begin{bmatrix}
Z_w & u_o \\
M_w & M_y
\end{bmatrix}
\begin{bmatrix}
-\frac{1}{I_{xx}} \Delta \theta \\
0
\end{bmatrix}
\begin{bmatrix}
I_{yy} & \frac{1}{I_{yy}} \Delta \phi \\
0 & N_v & N_r
\end{bmatrix}
\]

\[\Delta \chi(t) = \begin{bmatrix}
\Delta \omega \\
\Delta \dot{q} \\
\Delta \dot{\nu} \\
\Delta \dot{\nu}
\end{bmatrix}\]

\[\Delta \chi(t) = \begin{bmatrix}
\Delta \omega \\
\Delta \dot{q} \\
\Delta \dot{\nu} \\
\Delta \dot{\nu}
\end{bmatrix}\]

**Pitch-Yaw Coupling Due To Steady Roll Rate, \( p_o \)**

**Characteristic Polynomial**

\[
\Delta_{\text{pitch}}(s) = \left( s - Z_w \right) \left( s - M_y \right) \left( s - N_v \right) \left( s - N_r \right)
\]

\[
+ \frac{1}{I_{xx}} \left( I_{xx} - I_{yy} \right) \left( I_{xx} - I_{yy} \right) - \frac{1}{I_{yy}} \left( I_{yy} - I_{yy} \right) - \frac{1}{I_{yy}} \left( I_{yy} - I_{yy} \right)
\]

\[
- \frac{1}{I_{xx}} \left( I_{xx} - I_{xx} \right) \left( I_{xx} - I_{xx} \right) + \frac{1}{I_{yy}} \left( I_{yy} - I_{yy} \right) \left( I_{yy} - I_{yy} \right)
\]

- Coupling effect is proportional to \( p_o^2 \) and \( p_o^4 \)
- Effect on roots is independent of the sign of \( p_o \)
- Cannot use Evans’ root-locus rules with \( k = p_o^2 \), as \( k^2 \) also appears
- Can compute effect of \( p_o^2 \) on roots using MATLAB’s \( \text{eig} \)

\[
\Delta_{\text{pitch}}(s) = \left[ \Delta_{\text{pitch}}(s) \Delta_{\text{pitch}}(s) \right] + \frac{1}{I_{xx}} \left( I_{xx} - I_{xx} \right) \left( I_{xx} - I_{xx} \right) - \frac{1}{I_{yy}} \left( I_{yy} - I_{yy} \right) \left( I_{yy} - I_{yy} \right)
\]

**Thunderbird F-16 Barrel Roll**

http://www.youtube.com/watch?v=ovSOStlncbU
Effect of Steady Roll Rate, $p_o$, on Pitching and Yawing Roots

- Factor $\Delta_{\text{rolling}}(s)$ for various values of $p_o^2$
- $p_o^2 = \text{root locus gain, } k$
- Faster mode gets faster
- Slower mode gets slower and may become unstable

Steady Roll Rate, $p_o$, Effect Expressed by Root Locus or Parameter Plot

Parameter plot: variation of real and imaginary parts of roots vs. roll rate, $p_o$
Steady-State Response as Well as Stability is Affected by High Roll Rate

\[ f(v, w, p, q, r, \Delta \delta A, \Delta \delta E, \Delta \delta R)_{SS} = 0 \]

- Effects of steady roll rate on nonlinear equilibrium control response
  - Pitch-yaw coupling
  - "p jump" or "p acceleration"
- Multiple equilibria for same control settings
  - Up to 9 possible roll rates for one aileron setting
  - Sensitivity to elevator setting
    - Flight Dynamics, 7.3

The Butterfly Catastrophe*

\[ f_1(v, w, p, q, r, \Delta \delta A, \Delta \delta E, \Delta \delta R)_{SS} = 0 \]
\[ p_{SS} = f_2(v, w, q, r, \Delta \delta A, \Delta \delta E, \Delta \delta R)_{SS} \]
- Surface of equilibrium solutions for roll rate
- Possibility of an unrecoverable spin

* René Thom, 1974

after Mehra, Carroll, 1979
Tumbling and Spins

Tumbling, Spins, and Recovery

- Strong nonlinear effects
- Aircraft-specific control strategy for recovery
Wind Tunnel Spin Testing

- Sidney B. Gates, RAE: "The Spinning of Aeroplanes" (with L.W. Bryant, 1926), neutral and maneuver points, stick force per \( g \)
- Continued research on stalls and spins at NASA, USAF, and in many other countries

![Diagram of spin testing](image)

NASA Langley Spin Tunnel Testing

![Image of NASA Langley Spin Tunnel](image)

[http://www.youtube.com/watch?v=tQwMCml55Q0](http://www.youtube.com/watch?v=tQwMCml55Q0)
[http://www.youtube.com/watch?v=VUKTBUY1RlI](http://www.youtube.com/watch?v=VUKTBUY1RlI)
Tails with Negative Dihedral (Anhedral)

- Horizontal tail below wing's wake
- May have adverse effect on spin characteristics
- F-4 model test

![Graph showing the relationship between yaw coefficient ($C_n$) and spin rate ($\omega$) for different configurations.]

Yaw coefficient, $C_n$

Anti-spin moment

Pro-spin moment

Spin rate, $\omega/2V$

Yawing Moment at High Angle of Attack

- Dynamic as well as static effects, e.g., hysteresis
- Random asymmetric yawing moments (left or right)
  - generated by slender nose at zero sideslip angle
  - may exceed rudder control power

![Graph showing the relationship between yawing moment and angle of attack for different models.]

F-111

F-4

F-104

Vortex-induced side force on nose
Controlling Yawing Moment at High Angle of Attack

- Sucking, blowing, or movable strakes to control nose vortices
- X-29, F/A-18 HARV
- Vortex bursting effect on tail

Control Effectiveness at High Angle of Attack and Deflection Angle

- Assumption of Newtonian flow
Control at High Aerodynamic Angles

Supermaneuverability

- Means of forcing opponent to overshoot
- Pugachev’s Cobra maneuver, first done in Sukhoi Su-27
- Beneficial effect of thrust-vector control (X-31)
- Mongoose maneuver (X-31)
- Essentially low-speed maneuvers, not where you want to be in air combat (i.e., high energy-state)
Thrust Vector Control

Next Time:
Aeroelasticity and Fuel Slosh

Learning Objectives

- Aerodynamic effects of bending and torsion
- Modifications to aerodynamic coefficients
- Dynamic coupling
- Fuel shift and sloshing dynamics
Supplemental Material

Stall-Spin Studies of General Aviation Aircraft

http://www.youtube.com/watch?v=TmWB6oyJ9IE