Some Effects of Parameter Variations on the Lateral-Directional Stability of Aircraft

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Methods of analyzing the effects of aerodynamic uncertainties on the stability of lateral-directional motions are presented. Truncated and "residualized" reduced-order models of the motion are developed. Together with covariance analysis, these models provide simplified estimates of the probability of instability. A Space Shuttle entry flight condition is examined with this technique, and the predicted probability of open-loop instability is found to be low (<0.0001). Monte Carlo stability analysis illustrates that closed-loop control can increase the probability of instability as a consequence of uncertainty in the control derivatives.

Nomenclature

\[ b = \text{reference length} \]
\[ C_{l(1)}, C_{n(1)} = \text{dimensionless aerodynamic derivatives} \]
\[ c = \text{characteristic equation coefficient vector} \]
\[ F = \text{fundamental (stability) matrix} \]
\[ G = \text{control effects matrix} \]
\[ g = \text{gravitational acceleration magnitude} \]
\[ H_{b} = \text{body-to-stability-axis transformation matrix} \]
\[ I = \text{identity matrix} \]
\[ I_{a} = \text{inertia matrix} \]
\[ I_{x,y,z} = \text{moments and product of inertia} \]
\[ L_{(1)} = \text{dimensional roll stability and control derivatives} \]
\[ m = \text{standard deviation multiplier} \]
\[ N_{(1)} = \text{dimensional yaw stability and control derivatives} \]
\[ P = \text{parameter covariance matrix} \]
\[ p = \text{roll rate} \]
\[ p = \text{aerodynamic parameter vector} \]
\[ q = \text{dynamic pressure} \]
\[ r = \text{yaw rate} \]
\[ S = \text{reference area} \]
\[ s = \text{Laplace operator} \]
\[ V = \text{velocity magnitude} \]
\[ x = \text{state vector} \]
\[ Y_{(1)} = \text{dimensional side-force stability and control derivatives} \]
\[ \alpha = \text{real part of eigenvalue} \]
\[ \beta = \text{sideslip angle} \]
\[ \Delta(s) = \text{characteristic equation} \]
\[ \hat{b} = \text{control vector} \]
\[ \delta R, \delta A = \text{rudder and aileron deflections} \]
\[ \xi = \text{damping ratio} \]
\[ \Lambda = \text{eigenvalue covariance matrix} \]
\[ \lambda = \text{eigenvalue} \]
\[ \mu, \rho = \text{correlation coefficients} \]
\[ \sigma = \text{standard deviation} \]
\[ \tau = \text{time constant} \]

\[ \phi = \text{roll angle} \]
\[ \omega = \text{frequency; imaginary part of eigenvalue} \]

Subscripts and Superscripts (other than above)

\[ c = \text{command} \]
\[ D = \text{directional} \]
\[ DR = \text{Dutch roll} \]
\[ F = \text{fast dynamics} \]
\[ i = \text{index} \]
\[ L = \text{lateral} \]
\[ n = \text{natural} \]
\[ R = \text{roll mode} \]
\[ S = \text{spiral mode; also slow dynamics} \]

Introduction

The lateral-directional motions of conventional aircraft typically partition into Dutch roll, roll, and spiral modes; these modes provide "weather-cocking" directional stability, damped roll rate response, and a long-term tendency to maintain wings level or to "roll off" in a divergent spiral. When the dihedral effect is high and roll damping is low, the roll and spiral modes may coalesce in a single roll-spiral oscillation, or "lateral phugoid" mode. If this is combined with low directional stability and large adverse yaw due to aileron (or eleveon), the potential for piloting difficulties may become large. Uncertainties in aerodynamic parameters further increase the risks of mission degradation or failure on early flights of a new aircraft.

To a large extent, known deficiencies in flying qualities can be corrected by the aircraft's flight control system (FCS), but there is, of course, no guarantee that the FCS will properly account for unknown deficiencies. Closed-loop control tends to decrease the sensitivity of system performance to parameter variations, particularly if insensitivity is a design objective. Nevertheless, a given FCS may provide insensitivity to parameter variations of one type at the expense of increased sensitivity to variations of another type. For example, in minimizing performance degradation caused by stability derivative variations, a high-gain controller may inadvertently increase the sensitivity to control derivative variations. In such case, the statistics of the parameter variations, i.e., the probabilities that such variations will occur, are of paramount importance in achieving a "best" FCS design.

This paper presents an analysis of the effects of deterministic and stochastic parameter variations on the lateral-directional stability of an aircraft, using a Space Shuttle dynamic model for numerical examples. Linear dynamic models of lateral-directional motion are reviewed, with the specific objective of developing simple (reduced-order) models of the rigid-body motions. These models provide...
insight regarding the basic modes of motion, but more complex models are required to evaluate FCS effects. (One of the quandaries of modern aircraft FCS development is that the classical modes of motion tend to be masked by control compensation.) Higher-order models are used to illustrate the tradeoff between stability- and control-derivative sensitivity, and a technique for evaluating the probability of instability by Monte Carlo computation of eigenvalues is presented.

**Dynamic Equations**

The coupled dynamics of small lateral-directional perturbations typically are described by a fourth-order set of linear differential equations of the general form

\[
\Delta \dot{x} = F \Delta x + G \Delta \dot{\theta} \tag{1}
\]

where the motion variables are contained in \( \Delta x \),

\[
\Delta \dot{x}^T = (\Delta \dot{r} \ \Delta \theta \ \Delta \dot{\phi}) \tag{2}
\]

and the control variables are

\[
\Delta \dot{\theta}^T = (\Delta \dot{R} \ \Delta \dot{\phi}) \tag{3}
\]

Stability derivatives, inertial effects, and kinematic relationships are contained in \( F \). Neglecting unsteady aerodynamic effects, assuming that the flight path is horizontal, and considering only those aerodynamic side and lateral forces due to sideslip perturbation,

\[
F = \begin{bmatrix}
N_r & N_\delta & N_p & 0 \\
-1 & Y_p/V & 0 & g/V \\
L_r & L_\delta & L_p & 0 \\
0 & 0 & 1 & 0
\end{bmatrix} \tag{4}
\]

It is assumed that angles and angular rates are measured in stability axes, i.e., body axes which are nominally aligned with the velocity. The subscripted capital letters represent the sensitivities of specific forces and moments to state perturbations. The control effects matrix \( G \) is

\[
G = \begin{bmatrix}
N_{r\theta} & N_{\theta\theta} \\
0 & 0 \\
L_{r\theta} & L_{\theta\theta}
\end{bmatrix} \tag{5}
\]

Defining \( \Delta r \), \( \Delta \theta \), and \( \Delta \dot{R} \) as directional variables, and \( \Delta \rho \), \( \Delta \phi \), and \( \Delta \dot{\phi} \) as lateral variables, both \( F \) and \( G \) can be partitioned as

\[
F = \begin{bmatrix}
F_D & F_P^D \\
F_B & F_L^D
\end{bmatrix}, \quad G = \begin{bmatrix}
G_D & G_P^D \\
G_B & G_L^D
\end{bmatrix}
\]

The strength of lateral-directional coupling depends upon the relative magnitudes of the off-diagonal blocks, "Stability coupling," reflected in \( F \), and "control coupling," defined by \( G \), are distinctly separate phenomena; however, coupling in \( F \) almost certainly leads to coupled control response whether or not there is explicit coupling in \( G \). Control coupling affects stability only when there is external feedback, provided either by pilot loop closures or the flight control system.

There are two circumstances in which lateral and directional dynamics can be considered separately, i.e., in which decoupled reduced-order models produce reasonable approximations to stability and response characteristics. If the off-diagonal blocks are negligibly small, the fourth-order model can be decoupled. Directional stability is defined by \( F_D \), and lateral stability is defined by \( F_L \). This is a limiting case in nonsingular perturbation analysis, and the model reduction is referred to as "truncation.\" If \( F_D \) and \( F_B \) are not small but the time scales of yawing and rolling motions are widely separated, and if the faster mode is stable, then reduced-order models can be generated by "residualization." In this limiting case of singular perturbation analysis, the slow mode behaves as if the fast mode is always in steady-state equilibrium, and the fast mode "sees" the slow mode as a gradually changing bias. The fast mode affects the stability of the slow mode, but the reverse effect is negligible. Related approximate modes can be found in Refs. 1 to 3.

**Truncated Models**

When the off-diagonal blocks are small, the Dutch roll motions are approximated by the directional equation,

\[
\begin{bmatrix}
\Delta \dot{r} \\
\Delta \dot{\theta}
\end{bmatrix} = \begin{bmatrix}
N_r & N_\theta \\
-1 & Y_\theta/V
\end{bmatrix} \begin{bmatrix}
\Delta r \\
\Delta \theta
\end{bmatrix} + \begin{bmatrix}
N_{r\theta} \ \\
0
\end{bmatrix} \Delta \dot{R} \tag{8}
\]

and the roll and spiral motions are derived from the lateral equation,

\[
\begin{bmatrix}
\Delta \dot{\phi}
\end{bmatrix} = \begin{bmatrix}
L_p \\
1
\end{bmatrix} \begin{bmatrix}
\Delta \rho \\
\Delta \phi
\end{bmatrix} + \begin{bmatrix}
L_{\theta\phi} \ \\
0
\end{bmatrix} \Delta \dot{\theta} \tag{9}
\]

The roots of the corresponding second-order characteristic equations, \( \Delta(s) = \frac{s}{1 - s} - F = 0 \), govern the stability of the associated modes of motion. The Dutch roll natural frequency and damping ratio are approximated by

\[
\omega_{\text{DR}} = \left( N_{r\theta} + N_r Y_\theta/V \right)^{1/2} \tag{10}
\]

\[
\zeta_{\text{DR}} = -\frac{N_r + Y_\theta/V}{2\omega_{\text{DR}}} \tag{11}
\]

The roll mode time constant is \(-1/L_p\), and the spiral mode is neutrally stable, since its time constant is \( \infty \). From Eq. (4), this approximation is seen to be reasonable when dihedral effect \( L_p \) is small and \( V \) is large.

**Residualized Models**

A system with fast and slow modes can be partitioned as

\[
\begin{bmatrix}
\Delta x_f \\
\Delta x_s
\end{bmatrix} = \begin{bmatrix}
F_s & F_P^S \\
F_B^S & F_L^S
\end{bmatrix} \begin{bmatrix}
\Delta x_f \\
\Delta x_s
\end{bmatrix} + \begin{bmatrix}
G_s & G_P^S \\
G_B^S & G_L^S
\end{bmatrix} \begin{bmatrix}
\Delta \dot{\theta}_f \\
\Delta \dot{\theta}_s
\end{bmatrix} \tag{12}
\]

The fast mode can be approximated by

\[
\Delta \dot{x}_f = F \Delta x_f + F_P^S \Delta x_s + G_P^S \Delta \dot{\theta}_s + G_P^S \Delta \dot{\theta}_s \tag{13}
\]

where the second term is a slowly changing bias. The slow mode solution assumes that \( \Delta \dot{x}_s = 0 \). The lower row of Eq. (12) is solved algebraically for \( \Delta x_s \), and the result is substituted in the upper row of Eq. (12) to yield

\[
\begin{bmatrix}
\Delta x_s \\
\Delta x_f
\end{bmatrix} = \left( \begin{bmatrix}
F_s - F_P^S F_F^{-1} F_P^S \\
G_s - F_P^S F_F^{-1} G_P^S
\end{bmatrix} \right) \Delta \dot{\theta}_s + \begin{bmatrix}
F_L^S G_F \ \\
G_L^S \end{bmatrix} \Delta \dot{\theta}_f \tag{14}
\]
Three lateral-directional cases can be considered for residualization: 1) fast roll mode, neutral spiral mode, and slow Dutch roll mode; 2) fast roll mode, slow spiral mode, and slow Dutch roll mode; 3) slow lateral modes and fast directional mode.

In the first case, the roll angle can be neglected, and

\[
\begin{bmatrix}
F_x & F_y \\
F_y & F_z
\end{bmatrix} = \begin{bmatrix}
N_r & N_p & 0 \\
-1 & Y_p/V & 0 \\
-1 & Y_p/V & 0 \\
L_r & L_s & L_p
\end{bmatrix}
\]

(15)

Using Eqs. (13) and (14), the roll mode time constant is \(-1/L_p\); the Dutch roll natural frequency and damping ratio are

\[
\omega_{\text{DR}} = \left[ N_p + N_r Y_p/V - (L_r + L_s Y_p/V) N_p/L_p \right]^{1/2}
\]

(16)

\[
\xi_{\text{DR}} = -\left( N_r + Y_p/V - L_p N_p/L_p \right)/2\omega_{\text{DR}}
\]

(17)

In the second case,

\[
\begin{bmatrix}
F_x & F_y \\
F_y & F_z
\end{bmatrix} = \begin{bmatrix}
N_r & N_p & 0 \\
-1 & Y_p/V & 0 \\
0 & 0 & 0 \\
L_r & L_s & L_p
\end{bmatrix}
\]

The roll mode time constant is \(-1/L_p\), while the Dutch roll and spiral mode characteristics are determined by the roots of the third-order characteristic equation,

\[
\Delta(s) = s^3 - (N_r - L_s N_p/L_p + Y_p/V)s^2 + \left[(N_r - L_s N_p/L_p) Y_p/V + (g/V - N_p) L_r + L_s N_p/L_p \right]s + \left[(L_r N_p - L_s N_p) Y_p/V L_p = 0 \right] \]

(19)

When both roll and spiral motions are slower than directional motions,

\[
\begin{bmatrix}
F_x & F_y \\
F_y & F_z
\end{bmatrix} = \begin{bmatrix}
F_{\text{LR}} & F_{\text{LSR}} \\
F_{\text{LSR}} & F_{\text{SR}}
\end{bmatrix}
\]

(20)

Dutch roll characteristics are approximated by Eqs. (10) and (11), while roll-spiral modes are defined by

\[
\Delta(s) = s^2 + c_1 s + c_0 = 0
\]

(21)

with

\[
c_0 = (g/V) (L_r N_p - L_s N_p)/(N_p + Y_p/V)
\]

(22)

\[
c_1 = -L_s N_p (L_s + Y_p/V)/(N_p + Y_p/V)
\]

(23)

The roots of this equation may be real or complex, stable or unstable, depending on the sizes and magnitudes of the stability derivatives. A coupled roll-spiral oscillation is predicted when \((c_1^2 - 4c_0) < 0\).

Stability Effects of Parameter Variations

Deterministic and stochastic sensitivity of the stability characteristics can be estimated using the reduced-order models, subject to the validity of the assumptions described above. Analytical expressions which predict the probability of instability are readily defined for low-order models using assumptions of linearity and decoupling. For high-order systems and parameter variations which exceed the linear region of the root-parameter relationship, a Monte Carlo computation of the sort described in a later section may be required.

Each eigenvalue, \(\lambda_i = \alpha_i + j\omega_i, i = 1, \ldots, n\), can be written as a two-component real vector, \(\lambda^T = [\alpha_i, \omega_i]\), and it is a function of the characteristic equation coefficients \(c\):

\[
c^T = [c_0, c_1, \ldots, c_{n-2}, c_{n-1}]
\]

(24)

Expanding in a Taylor series about the nominal coefficient values,

\[
\lambda_i(c) = \lambda_i(c) + \frac{\partial \lambda_i}{\partial c} \big|_{c=c_0} (c-c_0) + \cdots
\]

(25)

The Jacobian matrix \(\partial \lambda_i/\partial c\) is the sensitivity of the eigenvalue to variations in the coefficients. Similarly, \(c\) can be expressed as a function of the parameters \(p\), which are the stability derivatives contained in Eq. (4):

\[
p^T = [L_p, L_s, L_r, N_p, N_s, N_r, Y_p]
\]

(26)

Expanding about the nominal parameter values,

\[
c(p) = c(p) + \frac{\partial c}{\partial p} \big|_{p=p_0} (p-p_0) + \cdots
\]

(27)

where \(\partial c/\partial p\) is the relevant Jacobian matrix. To first order, variations in the eigenvalue can be expressed as

\[
\Delta \lambda_i = \frac{\partial \lambda_i}{\partial c} \Delta c = \frac{\partial \lambda_i}{\partial p} \Delta p
\]

(28)

Assuming that \(p\) is a Gaussian variable with mean \(p_0\), the mean value of \(\lambda_i\) is

\[
\bar{\lambda}_i = \mathbb{E}[\lambda_i(c(p))] = \lambda_i(c(p_0))
\]

(29)

where \(\mathbb{E}[\cdot] \) is the expectation operator.

The statistical variation of \(\lambda_i\) about its mean is represented by the covariance matrix \(\Lambda_i\),

\[
\Lambda_i = \begin{bmatrix}
\sigma_{\alpha_i}^2 & \mu_i \sigma_{\alpha_i} \sigma_{\omega_i} \\
\mu_i \sigma_{\alpha_i} \sigma_{\omega_i} & \sigma_{\omega_i}^2
\end{bmatrix}
\]

(30)

with standard deviation \(\sigma_{\alpha_i}\) and \(\sigma_{\omega_i}\) as well as a correlation coefficient \(\mu_i\). Defining the parameter covariance matrix as

\[
P = \mathbb{E}(\Delta p \Delta p^T) = \begin{bmatrix}
\sigma_{\alpha_p}^2 & \rho_{12} \sigma_{\alpha_p} \sigma_{\alpha_r} & \cdots \\
\rho_{12} \sigma_{\alpha_p} \sigma_{\alpha_r} & \sigma_{\alpha_r}^2 & \cdots \\
\cdots & \cdots & \cdots
\end{bmatrix}
\]

(31)

the eigenvalue covariance \(\Lambda_i\) can be written as

\[
\Lambda_i = \mathbb{E}(\Delta \lambda_i \Delta \lambda_i^T) = \frac{\partial \lambda_i}{\partial c} \left[ \frac{\partial \lambda_i}{\partial p} \right]^T \Delta p \Delta p^T \frac{\partial \lambda_i}{\partial c}
\]

(32)

Equation (32) defines a "1-σ" ellipse about the mean eigenvalue, with principal axes skewed from the real and imaginary axes of the s-plane; the probability density function (pdf) of \(\lambda_i\) is

\[
\text{pdf}(\lambda_i) = (2\pi |\Lambda_i|)^{-1/2} \exp\left[-\frac{1}{2} (\lambda_i - \bar{\lambda}_i)^T \Lambda_i^{-1} (\lambda_i - \bar{\lambda}_i) \right]
\]

(33)
The probability of an instability in $\lambda_i$ is found by integrating Eq. (35) over all $\lambda$ and determining what portion of the integral lies in the right-half complex plane. If the coupling between real and imaginary variations in $\lambda_i$ is negligible ($\mu_i = 0$) or if $\lambda_i$ is real, then the probability that $\lambda_i$ will be driven to instability by parameter uncertainties is predicted by finding the value $m$ for which

$$\alpha_i + m\alpha_i = 0$$

(34)

and referring to standard tables for scalar Gaussian density functions. The probabilities associated with various degrees of instability, e.g., "times to double" of arbitrary value, are computed in similar fashion.

**Numerical Example**

**Models of Space Shuttle Dynamics**

Stability derivatives for the Space Shuttle flying at $M = 1.5$ on a return from orbit are derived from data in Ref. 8 and are given in Table 1. The mass of the vehicle is nominally 82,475 kg, corresponding to 181,450 lb, and the information is given in stability axes. $I_{xx}$, $I_{yy}$, and $I_{zz}$ are approximately $7.8 \times 10^3$, $5.9 \times 10^3$, and $1.5 \times 10^4$ slug-ft$^2$, respectively.

The fourth-order Dutch roll and roll-spiral eigenvalues of Eq. (4) are compared with reduced-order eigenvalues in Table 2. (Values in parentheses are real roll and spiral roots.) The truncated Dutch roll model is seen to be a reasonable approximation of the fourth-order Dutch roll, but the truncated lateral model fails to predict the roll-spiral oscillation. As a consequence of low roll damping, the lateral model is slower than the directional model. Case 3, with truncated Dutch roll and residualized roll-spiral modes, captures the natural frequencies but underestimates the damping of both modes.

Further definition of the Space Shuttle's unaugmented dynamics is given by lateral-directional eigenvectors and transient response to an eleven step input. The eigenvector magnitudes, which are synonymous with the "time vector" magnitudes of Ref. 3, are listed in Table 3 and are normalized to the sideslip angle response. It can be seen that rolling response is dominant for both modes of motion. The Dutch roll's "$\phi/\beta$" ratio is 8.7, and $\phi/\beta$ for the roll-spiral oscillation is ten times greater.

Roll response to aileron (or differential elevon) is the single most important lateral-directional control characteristic. The Space Shuttle has negative ("adverse") yaw response to elevon, indicating that the vehicle initially will yaw away from the commanded roll, causing a quasi-steady sideslip angle which "uncoordinates" the turn, as shown in Fig. 1. A 1-deg elevon deflection causes a quasi-steady roll rate of 4 deg/s which peaks after a lag of about 2 s. This is faster than the roll-mode time constant ($\tau_R = 1/\lambda_R$) predicted by the truncated model (Table 2), and strong modal coupling is apparent.

**Aerodynamic Uncertainties and Simplified Estimates of Their Effects**

In addition to aerodynamic coefficients, Ref. 8 provides estimates of 3-σ uncertainties and correlation coefficients for the static body-axis stability and control derivatives. These figures have been revised, but the preliminary values, shown in Table 4 as 1-σ values (i.e., one-third the original values), serve to demonstrate the analytical techniques presented here.

The dimensionless, body-axis coefficients are converted to dimensional, stability-axis derivatives in the conventional manner. For example, the roll- and yaw-moment effects of elevon are

$$\begin{bmatrix} L_{LA} \\ N_{LA} \end{bmatrix} = \hat{H} S_b \begin{bmatrix} C_{L_{BA}} \\ C_{N_{BA}} \end{bmatrix} q S_b$$

(35)

The lateral-directional inertia matrix is

$$I_b = \begin{bmatrix} I_{xx} & -I_{xz} \\ -I_{xz} & I_{zz} \end{bmatrix}$$

(36)

and the body-to-stability-axis transformation is

$$H_b^s = \begin{bmatrix} \cos \alpha_0 & \sin \alpha_0 \\ -\sin \alpha_0 & \cos \alpha_0 \end{bmatrix}$$

(37)

Then the aileron roll-yaw uncertainty covariance matrix $P$ transforms as

$$P_s = E \left[ \begin{bmatrix} \Delta L_{BA} \\ \Delta N_{BA} \end{bmatrix} \begin{bmatrix} \Delta L_{BA} \\ \Delta N_{BA} \end{bmatrix}^T \right]$$

(38)

where $\Delta( )$ connotes deviation from the mean value, and

$$E \left[ \begin{bmatrix} \Delta C_{L_{BA}} \\ \Delta C_{N_{BA}} \end{bmatrix} \begin{bmatrix} \Delta C_{L_{BA}} \\ \Delta C_{N_{BA}} \end{bmatrix}^T \right] = \begin{bmatrix} \sigma^2_{C_{L_{BA}}} & \rho \sigma_{C_{L_{BA}}} \sigma_{C_{N_{BA}}} \\ \rho \sigma_{C_{L_{BA}}} \sigma_{C_{N_{BA}}} & \sigma^2_{C_{N_{BA}}} \end{bmatrix}$$

(39)

### Table 1 Lateral-directional stability derivatives of the Space Shuttle*

| $L_1$ | 0.203 |
| $L_\alpha$ | -0.117 |
| $L_\phi$ | 0.587 |
| $L_{\beta}$ | 0.053 |
| $L_{\delta}$ | -0.594 |
| $L_{\delta \alpha}$ | 4.907 |
| $N_1$ | -0.090 |
| $N_\alpha$ | 0.008 |
| $N_\beta$ | 0.002 |
| $N_{\beta \delta}$ | 0.233 |
| $N_{\delta \alpha}$ | -0.397 |

* Mach number = 1.5, dynamic pressure = 1120 kg/m$^2$ (230 psf), angle of attack = 6.6 deg.

### Table 2 Comparison of lateral-directional eigenvalues of various Space Shuttle models

<table>
<thead>
<tr>
<th>Model</th>
<th>$\omega_{DR}$ (rad/s)</th>
<th>$\Omega_{DR}$ (rad/s)</th>
<th>$\Omega_{RS}$ (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fourth-order</td>
<td>0.749</td>
<td>0.310</td>
<td>0.139</td>
</tr>
<tr>
<td>Truncated models</td>
<td>0.773</td>
<td>0.134</td>
<td>-0.49</td>
</tr>
<tr>
<td>Residualized model: Case 1</td>
<td>0.233</td>
<td>0.397</td>
<td>-0.49</td>
</tr>
<tr>
<td>Residualized model: Case 3</td>
<td>0.773</td>
<td>0.134</td>
<td>0.166</td>
</tr>
</tbody>
</table>

### Table 3 Lateral-directional eigenvector magnitudes for the space shuttle example

<table>
<thead>
<tr>
<th>Model</th>
<th>Dutch roll mode</th>
<th>Roll-spiral mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_\alpha$, deg/s</td>
<td>0.78</td>
<td>2.01</td>
</tr>
<tr>
<td>$\Delta_{\phi}$, deg/s</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$\Delta_\phi$, deg/s</td>
<td>6.51</td>
<td>12.12</td>
</tr>
<tr>
<td>$\Delta_\beta$, deg</td>
<td>8.7</td>
<td>87.5</td>
</tr>
</tbody>
</table>
Estimates of stability-axis derivatives based on Table 4 are shown in Table 5. In spite of the rather large percentage uncertainties in body-axis \( C_{nA} \) and \( C_{nR} \), the net uncertainties in stability-axis \( N_R \) and \( N_N \) are of the same order as the remaining uncertainties. This is largely an effect of the correlations being not small (Table 5), they are neglected for simplicity below.

Standard deviations of Dutch roll and roll-spiral mode total damping (\( \alpha_n = -\omega_n \)) are estimated by applying Eq. (32) to the truncated and residualized models, Eqs. (8) and (20), using the data contained in Tables 1, 2, and 5. The value of \( m \) required for neutral stability [Eq. (34)] is calculated using the fourth-order eigenvalues as bases. For the Dutch roll mode, \( \sigma_n = 0.0899 \), and \( \omega_n = 0.232 \); hence, \( m = 26.1 \), and there is virtually no chance that parameter variation will drive this mode to instability. For the roll-spiral oscillation, \( \sigma_n = 0.0205 \), \( \omega_n = 0.117 \), and \( m = 5.7 \). A +5.7-\( \sigma \) parameter variation would cause instability, but the probability of this occurring is considerably less than 0.0001. Examining the contributions of individual parameter uncertainties to the roll-spiral \( \sigma_n \), the four most significant terms are ranked as follows: \( \sigma_{n,} \sigma_{n,} \sigma_{n,} \sigma_{n,} \).

Flight Control System Models

The Space Shuttle’s Early- and Late-Entry Digital Flight Control Systems have been simplified and modeled as continuous-time (“analog”) systems for analysis. Reaction control logic and structural-mode filters are eliminated, gains are held at constant values, and nonlinearities are linearized. The Early-Entry FCS (EFCS) shown in Fig. 2 uses rudder and reaction control thrusters to command stability-axis bank angle through kinematics, sideslip angle, and dihedral effect. With staged firing of thrusters, the reaction control logic approximates a quantized linear control moment which can be modeled, over a limited range, by increased values of \( N_{R,} \). Lateral hand controller inputs, \( \Delta D_{L,} \), are pre-filtered and subjected to proportional-integral compensation. The final low-pass filters prior to rudder and elevon commands, \( \Delta R_{L,} \) and \( \Delta R_{L,} \), are used to aggregate the effects of actuation delay and rate limiting, with nominal bandwidths of 10 rad/s. Turn coordination is provided by \( (\Delta r - \Delta g/V) \) feedback to controls. The EFCS uses five states to the closed-loop dynamic model described by Eq. (4); these correspond to the three low-pass filters and integral compensation.

The Late-Entry FCS (LFCS) shown in Fig. 3 uses differential elevons in conventional fashion, and it incorporates an aileron-rudder interconnect to account for adverse aileron yaw. There is proportional-integral compensation for both control surfaces, and the foot pedals, \( \Delta P_{A,} \), are enabled to aid turn coordination. Actuator delays are treated in the same way as above. The LFCS adds six states to the closed-loop dynamic model. The lateral acceleration feedback, \( \Delta a_{L,} \), is modeled as \( Y_{L,} \Delta a_{L,} \) for analysis. Additional details of the EFCS and LFCS models can be found in Ref. 10.
With nominal aerodynamic coefficients, both control modes provide substantial flying qualities improvements, as can be seen by comparing the lateral hand controller responses of Fig. 4 with Fig. 1. Roll rate response is essentially deadbeat, although the EFCS allows a slow increase that results from decreasing sideslip angle and negative dihedral effect. Sideslip response with the LFCS is two orders of magnitude smaller, allowing nearly ideal roll response.

The classical response modes described earlier are strongly affected by the flight control systems, as portrayed by the closed-loop eigenvalues and eigenvectors. Table 6 presents the Space Shuttle/EFCS eigenvalues together with the largest and next largest components of the corresponding eigenvectors. All modes are stable. The four lowest frequency modes are associated with the vehicle's rigid-body dynamics, while the faster modes involve the input and control surface states through EFCS dynamics. The first and fourth modes can be identified as spiral and roll modes, respectively, and there is no perceptible Dutch roll oscillation.

Table 7 presents similar information for the LFCS. The classical roll mode is essentially neutral, and the fourth (roll) mode is about twice as fast as the EFCS roll mode. The third mode is the Dutch roll, and the control system variables are dominant in four LFCS modes.

Thus it appears that Space Shuttle flying qualities benefit from the EFCS and LFCS when aerodynamic parameters are as predicted; however, the classical rigid-body modes can not be identified by their eigenvalues alone, and are difficult to identify from the eigenvectors. This poses a severe problem for the definition and application of parameter-based flying qualities criteria; results such as Fig. 4 suggest that analysis problems can be alleviated by using response-oriented criteria.

| Table 6 | Space shuttle eigenvalues and eigenvectors with the early-entry flight control system |
|-----------------|---------------------------------|------------------------|-----------------|
| \( \omega_0(\lambda) \), rad/s | \( \xi \) | Largest component | Second largest component |
| (-0.0007) | ... | Roll angle | Yaw rate |
| (-0.062) | ... | Roll angle | Roll rate |
| (-0.351) | ... | Roll angle | Roll rate |
| (-1.277) | ... | Roll rate | Roll angle |
| (-5) | ... | Command input | Elevon |
| (-8.647) | ... | Rudder | Elevon |
| 10.32 | 0.502 | Elevon | Roll rate |

| Table 7 | Space shuttle eigenvalues and eigenvectors with the late-entry flight control system |
|-----------------|---------------------------------|------------------------|-----------------|
| \( \omega_0(\lambda) \), rad/s | \( \xi \) | Largest component | Second largest component |
| (+2.6 \times 10^{-5}) | ... | Roll angle | Yaw rate |
| 0.166 | 0.924 | Roll angle | Rudder integrator |
| 2.164 | 0.552 | Rudder | Roll rate |
| (-2.641) | ... | Roll rate | Rudder |
| (-5) | ... | Elevon | Roll rate |
| (-5.877) | ... | Elevon | Rudder |
| 7.575 | 0.956 | Rudder | Elevon |

**Effects of Parameter Variations**

Deterministic Stability Boundaries

Deterministic stability boundaries for open- and closed-loop models are defined by varying the aerodynamic derivatives, computing the eigenvalues, and cross-plotting to define the regions of stability and instability as functions of the derivatives. This approach identifies the percentage variation needed to cause instability, but it gives no indication of the probability of exceeding a stability boundary.

\( L_p \) and \( N_p \) variations of ±100% affect unaugmented airframe stability as presented in Fig. 5. Flying qualities improve as \( N_p \) increases and \( L_p \) decreases, with separate roll and spiral modes reappearing for variations of 30% to 40% in either variable. The roll-spiral mode is driven to instability by comparable variations of opposite sign.

Although not shown, \( L_p \) and \( N_p \) variations of -45% and +30% induce an unstable roll-spiral mode, while \( N_p \) must increase by 80% to cause a spiral instability. Combining -90% \( N_p \) variation with +90% \( N_p \), variation causes Dutch roll instability. No instabilities are caused by ±100% variations of \( L_p \).

With a minor exception, ±100% variations in these six stability derivatives introduce no new instabilities when either the EFCS or LFCS is engaged. The EFCS has an unstable roll-spiral mode when \( N_p \) has dropped to zero and \( L_p \) is doubled. The two control systems provide from 5 to 20 times the damping of the unaugmented vehicle; hence, 100% variations in \( L_p \), \( L_r \), \( N_p \), and \( N_r \) have negligible effect. However, the reduced sensitivity to stability-derivative variations is ac-
Fig. 7 Effects of $L_{bA}$ and $N_{bA}$ on Space Shuttle stability, early-entry FCS.

Table 8 Probability of instability due to aerodynamic variations

<table>
<thead>
<tr>
<th>Case (no. of trials)</th>
<th>$\tau$(stable), s</th>
<th>$-\tau$(unstable), s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5-10</td>
<td>10-100</td>
</tr>
<tr>
<td>FCS off (1000)</td>
<td>70</td>
<td>30</td>
</tr>
<tr>
<td>EFCS$^a$ (1000)</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>LFCS$^a$ (1000)</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>EFCS$^b$ (100)</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>LFCS$^c$ (100)</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

$^a$Actuator bandwidth = 10 rad/s. $^b$Actuator bandwidth = 1 rad/s.

Conclusion

Models for examining the effects of aerodynamic parameter variations on lateral-directional stability have been developed and applied to a Space Shuttle example. Conditions for using truncated and residualized reduced-order models are presented, and their utility for characterizing eigenvalue sensitivity is shown. Covariance analysis is used to predict the probability of instability with these simple models. The probability of instability in higher-order dynamic systems, as introduced by flight control compensation, is evaluated by Monte Carlo computation of eigenvalues, a computer-intensive but general approach. With assumed values of parameter uncertainty, the example's stability is found to be relatively tolerant to variations in aerodynamic derivatives. Feedback control provides insensitivity to stability derivative variations, but "tight" control increases sensitivity to control derivative variations; hence, the probability of instability due to uncertainties in control effects is heightened in this example.
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References

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